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NONDESTRUCTIVE TECHNIQUES FOR DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF MATERIALS BY THE INSTANTANEOUS-HEAT-SOURCE METHOD

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Using the method of a linear instantaneous heat source, we consider nondestructive techniques for determining the thermophysical characteristics of materials. A procedure for estimating errors in the techniques suggested is given.

At the present time in the scientific literature a considerable body of works are available that are concerned with methods of nondestructive determination and control of the thermal properties of materials on the basis of the action of an instantaneous heat source [1-4]. In [4], two-dimensional axisymmetric nonstationary heat conduction problems are considered, such as those of thin circular and annular heat sources of zero heat capacity acting in the plane of contact between two bodies with different thermophysical characteristics (TPC).

Below we consider methods based on application of a linear instantaneous heat source with zero heat capacity that complement the range of problems considered in [4]. The merits of the methods suggested are the comparative ease of their technical implementation and the small time needed for performing an experiment.

Let us now consider a system consisting of two semi-infinite bodies with different thermophysical characteristics and of a source in the form of a straight line located in the plane of their contact. At the initial time instant the source liberates an amount of heat Q instantly and uniformly over the entire length (on a per unit length basis). It is required to find the temperature field in the plane of contact of the bodies at all the subsequent time instants. We will assume that the heat capacity and the region of the heat source localization are negligibly small compared to the heat capacity and dimensions of the two semi-infinite materials, which are in ideal thermal contact with the linear heat source. We will place the coordinate origin on the boundary plane and the linear source along the y axis. The initial temperature of the system will be taken as zero.

We have

$$\frac{\partial T(x, z, \tau)}{\partial \tau} = a_1 \left(\frac{\partial^2 T(x, z, \tau)}{\partial x^2} + \frac{\partial^2 T(x, z, \tau)}{\partial z^2} \right)$$

$$(z < 0, \ -\infty < x < \infty, \ \tau \geq 0);$$

$$\frac{\partial T(x, z, \tau)}{\partial \tau} = a_2 \left(\frac{\partial^2 T(x, z, \tau)}{\partial x^2} + \frac{\partial^2 T(x, z, \tau)}{\partial z^2} \right) + \frac{a_2}{\lambda_2} f(x, z, \tau) \tag{1}$$

$$(z > 0, \ -\infty < x < \infty, \ \tau \geq 0)$$

under the conditions

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$$T(x, 0_+, \tau) = T(x, {}_-0, \tau), \quad T(x, z, 0) = 0,$$

$$\lambda_1 \frac{\partial T}{\partial z}\bigg|_{z={}_-0} = \lambda_2 \frac{\partial T}{\partial z}\bigg|_{z=0_+},$$

$$T \to 0 \quad |z| \to \infty,$$

$$f(x, z, \tau) = \frac{Q}{\varepsilon}\delta(x)\,\delta(\tau)\,\chi(z\,[0, \varepsilon]),$$

where $\delta(\xi)$ is the delta-function; $\chi(z[0, \varepsilon])$ is the characteristic function of the interval $[0, \varepsilon]$, i.e.,

$$\chi(z\,[0, \varepsilon]) = \begin{cases} 1, & z \in [0, \varepsilon], \\ 0, & z \notin [0, \varepsilon]. \end{cases}$$

The solution of system (1) in the boundary plane with $\varepsilon \to 0$ has the form

$$T(x, \tau) = \frac{Q}{2\pi\tau\,(\lambda_1^2 - \lambda_2^2)}\left\{\lambda_1 \exp\left(-\frac{x^2}{4a_1\tau}\right) - \lambda_2 \exp\left(-\frac{x^2}{4a_2\tau}\right) + \right.$$
$$+ \frac{x}{2\tau}\exp\left(-\frac{x^2}{4D\tau}\right)\left[\lambda_1\left(\frac{1}{a_1} - \frac{1}{D}\right)\int_0^x \exp\left[-\left(\frac{1}{a_1} - \frac{1}{D}\right)\frac{\xi^2}{4\tau}\right]d\xi - \right.$$
$$\left.\left. - \lambda_2\left(\frac{1}{a_2} - \frac{1}{D}\right)\int_0^x \exp\left[-\left(\frac{1}{a_2} - \frac{1}{D}\right)\frac{\xi^2}{4\tau}\right]d\xi\right]\right\}, \tag{2}$$

where

$$D = \left(\lambda_1^2 - \lambda_2^2\right) \Big/ \left(\frac{\lambda_1^2}{a_1} - \frac{\lambda_2^2}{a_2}\right) \quad \text{when} \quad \frac{\lambda_1^2}{a_1} \neq \frac{\lambda_2^2}{a_2};$$

Q is the amount of heat liberated instantaneously by the source over a section of unit length (J/m); x is the distance reckoned along a normal from the line of source action (m); τ is the time reckoned from the instant of action of a heat pulse (sec); λ_1 and λ_2 are the thermal conductivities (W/(m·K)); a_1 and a_2 are the thermal diffusivities (m^2/sec) of the investigated and standard materials.

Expression (2) is rather complex for direct determination of thermophysical characteristics. Expanding the right-hand side of formula (2) into a series and discarding terms higher than the third power with respect to x, we obtain

$$T(x, \tau) = \frac{Q}{2\pi(\lambda_1 + \lambda_2)\tau}\left(1 - \frac{\dfrac{\lambda_1}{a_1} + \dfrac{\lambda_2}{a_2}}{\lambda_1 + \lambda_2}\frac{x^2}{4\tau}\right). \tag{3}$$

In order to ascertain the adequacy of solutions (2) and (3), as well as the admissible limits for the application of approximate solution (3), we calculated on a computer the temperature fields for different values of the coefficients λ and a. The results obtained confirm that as the value of x decreases and that of τ increases, the difference between the temperatures calculated from expressions (2) and (3) tends to zero, whereas at $\tau = 6$ sec it becomes rather small and comparable with random errors of measurements (for $x \leq 2\cdot10^{-7}$ m). The value of τ can be limited to 10–12 sec, since at larger values of time the temperature depends virtually on the magnitude of $(\lambda_1 + \lambda_2)$. Expression (3) is valid if $(x^2/4a_{\min}\tau) \ll 1$, i.e., at large Fourier numbers, $\mathrm{Fo} = 4a_{\min}\tau/x^2$, which makes it possible to obtain an approximation accurate to $(1/\mathrm{Fo})^2$.

Thus, formula (3) can be used in engineering calculations of temperature fields by the method of a linear instantaneous heat source at small values of x.

It is seen from the structure of function (3) that, using some other material with known thermal properties (standard material), we can determine the thermophysical characteristics of the material investigated by recording the temperature in the plane of their separation at prescribed time instants.

Solution (2) for the temperature field on the line of heat source action ($x = 0$) has the form

$$T(0, \tau) = \frac{Q}{2\pi(\lambda_1 + \lambda_2)\tau}. \tag{4}$$

From this, assuming the coefficient λ_2 to be known, we obtain a formula for the thermal conductivity coefficient of the material under investigation:

$$\lambda_1 = \frac{Q}{2\pi\tau\, T(0, \tau)} - \lambda_2, \tag{5}$$

where τ is the time instant at which we perform temperature recording $T(0, \tau)$.

For heat pulses of finite duration τ^*

$$\lambda_1 = \frac{Q}{2\pi\tau^*\, T(0, \tau)} \ln\left(\frac{\tau}{\tau - \tau^*}\right) - \lambda_2. \tag{6}$$

If we measure the temperature at a point at distance x from the linear heat source (in the plane of contact between the material investigated and the material with the known thermophysical characteristics) at time instants τ_1 and τ_2, then with allowance for Eq. (3) we obtain

$$\lambda_1 = M - \lambda_2, \quad a_1 = \lambda_1/(KM - \lambda_2/a_2), \tag{7}$$

where

$$K = \frac{4(A_1 - A_2)}{x^2(A_1/\tau_2 - A_2/\tau_1)};$$

$$M = (1 - Kx^2/4\tau_1)/A_1; \quad A_i = 2\pi\tau_i T_i/Q, \quad i = 1, 2.$$

By measuring at a prescribed time instant τ_1 the temperature at two points of the boundary plane of the investigated and standard materials, one of which lies on the line of heat source action, while the other at the distance x_1 from the source, with account for solution (3) we obtain the formulas [5]:

$$\lambda_1 = \frac{Q}{2\pi\tau_1\, T(0, \tau_1)} - \lambda_2; \tag{8}$$

$$a_1 = \lambda_1 \Bigg/ \left\{\frac{2Q\,[T_1(0, \tau_1) - T_1(x_1, \tau_1)]}{\pi x_1^2 T_1^2(0, \tau_1)} - \frac{\lambda_2}{a_2}\right\}. \tag{9}$$

To increase the accuracy in determining the thermophysical characteristics of the material investigated after the heat pulse supply, we record the temperature at a prescribed time instant τ_1 at the same points of the boundary plane ($x = 0$ and $x = x_1$) for a system consisting of two standard specimens. Then the formulas for λ_1 and a_1 of the material investigated have the form

$$\lambda_1 = \lambda_2\left(\frac{2}{N} - 1\right), \quad a_1 = a_2\left(\frac{2 - N}{2M - N}\right), \tag{10}$$

where

$$M = \frac{[T_1(0, \tau_1) - T_1(x_1, \tau_1)]\, T_2(0, \tau_1)}{[T_2(0, \tau_1) - T_2(x_1, \tau_1)]\, T_1(0, \tau_1)}; \quad N = \frac{T_1(0, \tau_1)}{T_2(0, \tau_1)};$$

$T_1(0, \tau_1)$, $T_1(x_1, \tau_1)$ are the temperatures measured in the plane of contact of the investigated and standard materials; $T_2(0, \tau_1)$, $T_2(x_1, \tau_1)$ are the temperatures measured in the contact plane of the system of two standard specimens.

Thus, by performing preliminary measurements for a system of two standard materials it is possible to exclude the effect of systematic errors in measurements of Q and x on the accuracy of determining thermophysical characteristics.

If in the considered "investigated material–standard" system we replace the material with the known thermophysical characteristics by a heat insulator, then on the basis of solution (2) the temperature on the surface of the investigated specimen under the action of an instantaneous linear heat source with $\tau \to 0$ and $\lambda_2 = 0$, $a_2 = 0$ will be described by the relation [1]

$$T(x, \tau) = \frac{Q}{2\pi\lambda_1\tau} \exp\left(-\frac{x^2}{4a_1\tau}\right). \tag{11}$$

Recording the time instant τ_1 after the heat pulse supply, when the relationship between the temperatures at two points x_1 and x_2 $(0 < x_1 < x_2)$ attains a certain specified value

$$T(x_1, \tau_1) = nT(x_2, \tau_1), \; n > 1,$$

we obtain expressions for the desired coefficients [6]:

$$a_1 = \frac{x_2^2 - x_1^2}{4\tau_1 \ln n}, \quad \lambda_1 = \frac{Q}{2\pi\, T(x_1, \tau_1)\, \tau_1} \exp\left(-\frac{x_1^2}{4a_1\tau_1}\right). \tag{12}$$

If measurements of temperature after the heat pulse supply are carried out on the line of heating, $T(0, \tau)$, and at a given distance from it, $T(x_1, \tau)$ and the time instant τ_1 is recorded under the condition

$$T(x_1, \tau_1) = T(0, \tau_1) - T(x_1, \tau_1), \tag{13}$$

then the thermophysical characteristics of the material investigated with allowance for Eq. (11) can be determined by formulas of [7]:

$$a_1 = -\frac{x_1^2}{4 \ln 0.5}, \quad \lambda_1 = \frac{Q\,[1 + \exp(\ln 0.5)]}{2\pi\tau_1\,[T(0, \tau_1) - T(x_1, \tau_1)]}. \tag{14}$$

To increase the accuracy in determining thermophysical characteristics, it is necessary first to conduct the above measurements of the temperatures $T(0, \tau)$ and $T(x_1, \tau)$ on the material with the known characteristics and to record the characteristic time τ_0 that corresponds to equality between the excess temperature $T(x_1, \tau_0)$ at a point at a prescribed distance x_1 from the line of heating and the difference of the excess temperatures $T(0, \tau_0) - T(x_1, \tau_0)$. Then we obtain [8]

$$a_1 = a_2 \frac{\tau_0}{\tau_1}, \quad \lambda_1 = \lambda_2 \frac{[T(0, \tau_0) - T(x_1, \tau_0)]\, \tau_0}{[T(0, \tau_1) - T(x_1, \tau_1)]\, \tau_1}, \tag{15}$$

where λ_2 and a_2 are the coefficients of thermal conductivity and thermal diffusivity of the material with the known characteristics.

This type of approach allows one to eliminate systematic errors in measurements to a large degree and to avoid direct and indirect determinations of the values of Q and x.

In conducting thermophysical measurements, temperature sensors frequently experience the effect of periodic random electric and magnetic fields. As a result, the desired output signal of the sensor, for example, the emf of a thermocouple, can be superimposed on random interference. Therefore, when a temperature sensor is exposed to the action of random periodic interferences, it is worthwhile to develop methods of nondestructive control of the thermophysical characteristics for those materials, in which the recording is made not of the absolute values of temperature, but rather of its integrated values.

Let us determine the integrated value of the temperature described by formula (4) over the time interval $\Delta\tau = \tau_2 - \tau_1$:

$$S_1 = \int\limits_{\tau_1}^{\tau_2} T(0, \tau)\, d\tau = \frac{Q}{2\pi(\lambda_1 + \lambda_2)} \ln \frac{\tau_2}{\tau_1} \tag{16}$$

and obtain an expression for the thermal conductivity coefficient

$$\lambda_1 = \frac{Q}{2\pi S} \ln \frac{\tau_2}{\tau_1} - \lambda_2. \tag{17}$$

For heat pulses of finite duration τ^*

$$S = \frac{Q}{2\pi(\lambda_1 + \lambda_2)\tau^*} \left(\tau_2 \ln \frac{\tau_2}{\tau_2 - \tau^*} - \tau_1 \ln \frac{\tau_1}{\tau_1 - \tau^*} + \tau^* \ln \frac{\tau_2 - \tau^*}{\tau_1 - \tau^*} \right);$$

$$\lambda_1 = \frac{QK}{2\pi\tau^* S} - \lambda_2, \tag{18}$$

where

$$K = \tau_2 \ln \frac{\tau_2}{\tau_2 - \tau^*} - \tau_1 \ln \frac{\tau_1}{\tau_1 - \tau^*} + \tau^* \ln \frac{\tau_2 - \tau^*}{\tau_1 - \tau^*}.$$

The thermal conductivity coefficient can be determined from formulas (16) and (17) in two ways: 1) we assign in advance the values for the time instants τ_1 and τ_2 with subsequent determination of the value of S; 2) we assign the values of τ_1 and S with subsequent determination of the time instant τ_2.

If under the action of an instantaneous heat source we first perform integration over the same time interval $[\tau_1, \tau_2]$ for a system of two specimens with the same known thermophysical characteristics (standard-standard), then

$$S_2 = \frac{Q}{4\pi\lambda_2} \ln \frac{\tau_2}{\tau_1}. \tag{19}$$

From formulas (16) and (19) it follows that

$$\lambda_1 = \left(2 \frac{S_2}{S_1} - 1 \right) \lambda_2, \tag{20}$$

where λ_1 and λ_2 are the thermal conductivity coefficients of the investigated and standard specimens, respectively; S_1 and S_2 are the integrated values of temperatures obtained in the regimes of measurement and standardization over the time intervals $[\tau_1, \tau_2]$.

We also obtained formulas for the coefficients of thermal conductivity and thermal diffusivity when the integrated values of the temperature S [9] are measured: for the time interval $\Delta\tau = \tau_2 - \tau_1$ at a point located at distance x_1 from the heat source line; for the time interval $\Delta\tau_1 = \tau_1 - \tau_0$ and $\Delta\tau_2 = \tau_2 - \tau_0$; for the time intervals $\Delta\tau = \tau - \tau_0$ at two points, one of which is located on the line of action of the heat source, while the other is at distance x_1 from it, and when the relationships between the integrated values of the temperatures on the line of action of the heat source and at the prescribed distance from it in the "standard–standard" and "investigated material–standard" systems attain a certain preassigned value.

Analysis of the proposed formulas for the thermophysical characteristics of materials with recorded absolute and integrated values of temperatures shows that the thermophysical characteristics of the materials are determined by a system of equations:

$$A_1 = \frac{1}{\lambda_1 + \lambda_2}\left(1 - \frac{\frac{\lambda_1}{a_1} + \frac{\lambda_2}{a_2}}{\lambda_1 + \lambda_2} B_1\right) + C_1 ;$$

$$A_2 = \frac{1}{\lambda_1 + \lambda_2}\left(1 - \frac{\frac{\lambda_1}{a_1} + \frac{\lambda_2}{a_2}}{\lambda_1 + \lambda_2} B_2\right) + C_2 , \tag{21}$$

where A_i and B_i ($i = 1, 2$) are independent of λ and a; C_i depend on λ and a; $B_i > 0$, while the calculated values for λ and a are determined at $C_1 = C_2 = 0$:

$$\lambda_1 = \frac{B_2 - B_1}{B_2 A_1 - B_1 A_2} - \lambda_2 ; \quad a_1 = \frac{\lambda_1}{(\lambda_1 + \lambda_2) K - \frac{\lambda_2}{a_2}} , \tag{22}$$

where

$$K = (\lambda_1 + \lambda_2) \frac{A_1 - A_2}{B_2 - B_1} .$$

To estimate the errors in the obtained values for the thermophysical characteristics of the materials, we composed an auxiliary system:

$$A_1 = \frac{1}{\lambda_1 + \lambda_2}\left(1 - \frac{\frac{\lambda_1}{a_1} + \frac{\lambda_2}{a_2}}{\lambda_1 + \lambda_2} B_1\right) + C_1 h ;$$

$$A_2 = \frac{1}{\lambda_1 + \lambda_2}\left(1 - \frac{\frac{\lambda_1}{a_1} + \frac{\lambda_2}{a_2}}{\lambda_1 + \lambda_2} B_2\right) + C_2 h , \quad h \in [0, 1] , \tag{23}$$

from which system (21) is obtained at $h = 1$ and formulas (22) for the thermophysical characteristics are obtained when $h = 0$.

The absolute errors in determining the coefficients λ and a can be estimated by the differential of the functions $\lambda(h)$ and $a(h)$ at the point $h = 0$ (an increment of the argument h is equal to 1). We obtain

$$|\Delta\lambda| \le \frac{B_2 |C_1| + B_1 |C_2|}{|B_2 - B_1|} (\lambda_1 + \lambda_2)^2 ; \tag{24}$$

$$|\Delta a| \le \frac{a}{\lambda} (|1 - aK| |\Delta\lambda| + a |\lambda_1 + \lambda_2| |\Delta K|) ,$$

where

$$|\Delta K| \le \frac{A_2 |C_1| + A_1 |C_2|}{|B_2 - B_1|} (\lambda_1 + \lambda_2)^2 .$$

The coefficient C_i ($i = 1, 2$) is a function of the difference ΔT_i between the actual (Eq. (2)) and approximated (Eq. (3)) temperatures. As the absolute values of C_i or ΔT_i decrease, the accuracy of the procedures suggested increases.

We carried out an experimental determination of the thermophysical characteristics of a number of heat-insulating materials (organic glass PMMA, foam plastics PS-I, PPU-104B). Measurements were performed by means of an automatic device [10] based on a "Vektor" personal computer using the method of a linear instantaneous heat source with recording, at a prescribed time instant, of the temperature at two points on the plane of contact between the investigated and standard specimens.

As a material with known characteristics, we used foam plastic PS-4 ($\lambda_2 = 0.0435$ W/(m·K); $a_2 = 4.74 \cdot 10^{-7}$ m^2/sec). The relative errors in determining the coefficients of thermal conductivity and thermal diffusivity amount to 8.1–8.9 and 10.7–12.3%, respectively.

Thus, using the method of a linear instantaneous heat source with recording of the absolute and integrated values of temperature, we have considered the developed techniques for determining the thermophysical characteristics of materials without destruction of their integrity and suggested a procedure for estimating the errors of the proposed techniques and presented the results of experimental determination of the thermal conductivity and thermal diffusivity of some heat insulating materials.

NOTATION

x, y, z, space coordinates; τ, time; λ, thermal conductivity coefficient; a, thermal diffusivity coefficient; Q, amount of heat per unit length; S, integrated value of temperature; Fo, Fourier number; T, temperature.

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